Unit 1 – Operations Review

(3 weeks, September)

<u>Common Core State Standards Addressed:</u>

5-NF.1: Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*
5-NF.4: Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
5-NF.7: Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.
6-EE.1: Write and evaluate numerical expressions involving whole-number exponents.
6-EE.3: Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y*.
7-NS.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
7-NS.2: Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
7-NS.3: Solve real-world and mathematical problems involving the four operations with rational numbers.

<u>Student Friendly Learning Targets (In order of teaching):</u>

- I can use order of operations.
- I can add and subtract rational numbers.
- I can multiply rational numbers.
- I can divide rational numbers.
- I can add and subtract integers.
- I can multiply and divide integers.

<u>Vocabulary:</u>

expression, order of operations, grouping symbols, exponents, fraction, mixed number, improper fraction, numerator, denominator, integer, opposite, zero pair

<u>Materials and/or Technology Needed:</u>

calculator

<u>Instructional Strategies:</u>

Information from all Learning Targets can be found at Ohio's Model Curricula: http://education.ohio.gov/Topics/Academic-Content-Standards/Mathematics and KCTM's Flip Book (which encompasses several states' information including Ohio): http://www.katm.org/baker/pages/common-core-resources.php

- **I can use order of operations.**

(From Ohio's Model Curricula) The skills of reading, writing and evaluating expressions are essential for future work with expressions and equations, and are a Critical Area of Focus for Grade 6. In earlier grades, students added grouping symbols ( ) to reduce ambiguity when solving equations. Now the focus is on using ( ) to denote terms in an expression or equation. Students should now focus on what terms are to be solved first rather than invoking the PEMDAS rule. Likewise, the division symbol (3 ÷ 5) was used and should now be replaced with a fraction bar ($\frac{3}{5}$). Less confusion will occur as students write algebraic expressions and equations if *x* represents only variables and not multiplication. The use of a dot (•) or parentheses between number terms is preferred.

Provide opportunities for students to write expressions for numerical and real-world situations. Write multiple statements that represent a given algebraic expression. For example, the expression $x - 10$ could be written as "ten less than a number," "a number minus ten," "the temperature fell ten degrees,"', "I scored ten fewer points than my brother," etc. Students should also read an algebraic expression and write a statement.

Through modeling, encourage students to use proper mathematical vocabulary when discussing terms, factors, coefficients, etc.

Provide expressions and formulas to students, along with values for the variables so students can evaluate the expression. Evaluate expressions using the order of operations with and without parentheses. Include whole-number exponents, fractions, decimals, etc. Provide a model that shows step-by-step thinking when simplifying an expression. This demonstrates how two lines of work maintain equivalent algebraic expressions and establishes the need to have a way to review and justify thinking.

Provide a variety of expressions and problem situations for students to practice and deepen their skills. Start with simple expressions to evaluate and move to more complex expressions. Likewise start with simple whole numbers and move to fractions and decimal numbers. The use of negatives and positives should mirror the level of introduction in Grade 6 The Number System; students are developing the concept and not generalizing operation rules.

The use of technology can assist in the exploration of the meaning of expressions. Many calculators will allow you to store a value for a variable and then use the variable in expressions. This enables the student to discover how the calculator deals with expressions like x^2, $5x$, xy, and $2(x + 5)$.

<u>Common Misconceptions from Ohio's Model Curricula:</u>
Many of the misconceptions when dealing with expressions stem from the misunderstanding/reading of the expression. For example, knowing the operations that are being referenced with notation like, x^3, 4*x*, 3(*x* + 2*y*) is critical. The fact that x^3 means *x*•*x*•*x*, means *x* times *x* times *x*, not 3*x* or 3 times *x*; 4*x* means 4 times *x* or *x*+*x*+*x*+*x*, not forty-something. When evaluating 4*x* when *x* = 7, substitution does not result in the expression meaning 47. Use of the "*x*" notation as both the variable and the operation of multiplication can complicate this understanding.

- **I can add and subtract rational numbers.**

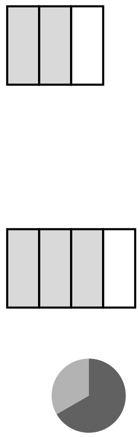
(From Ohio's Model Curricula) To add or subtract fractions with unlike denominators, students use their understanding of equivalent fractions to create fractions with the same denominators. Start with problems that require the changing of one of the fractions and progress to changing both fractions. Allow students to add and subtract fractions using different strategies such as number lines, area models, fraction bars or strips. Have students share their strategies and discuss commonalities in them.

Students need to develop the understanding that when adding or subtracting fractions, the fractions must refer to the same whole. Any models used must refer to the same whole. Students may find that a circular model might not be the best model when adding or subtracting fractions.

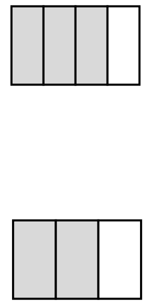
As with solving word problems with whole number operations, regularly present word problems involving addition or subtraction of fractions. The concept of adding or subtracting fractions with unlike denominators will develop through solving problems. Mental computations and estimation strategies should be used to determine the reasonableness of answers. Students need to prove or disprove whether an answer provided for a problem is reasonable.

Estimation is about getting useful answers, it is not about getting the right answer. It is important for students to learn which strategy to use for estimation. Students need to think about what might be a close answer.

<u>Common Misconceptions from Ohio's Model Curricula:</u>
Students often mix models when adding, subtracting or comparing fractions. Students will use a circle for thirds and a rectangle for fourths when comparing fractions with thirds and fourths. Remind students that the representations need to be from the same whole models with the same shape and size.



These models of fractions are difficult to compare because the size of the whole is not the same for all representations

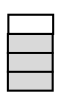
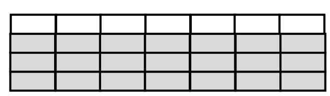


These models of fractions use the same size rectangle to represent the whole unit and are therefore much easier to compare fractions.

- **I can multiply rational numbers.**
- **I can divide rational numbers.**

(From Ohio's Model Curricula for both) Connect the meaning of multiplication and division of fractions with whole-number multiplication and division. Consider area models of multiplication and both sharing and measuring models for division.

Ask questions such as, "What does 2 × 3 mean?" and "What does 12 ÷ 3 mean?" Then, follow with questions for multiplication with fractions, such as, "What does $\frac{3}{4} \times \frac{1}{3}$ mean?" "What does $\frac{3}{4}\times 7$ mean?" (7 sets of $\frac{3}{4}$) and What does $7 \times \frac{3}{4}$mean?" ($\frac{3}{4}$ of a set of 7)
The meaning of $4 \div \frac{1}{2}$ (how many $\frac{1}{2}$ are in 4) and $\frac{1}{2} \div 4$(how many groups of 4 are in $\frac{1}{2}$) also should be illustrated with models or drawings like:

$\frac{3}{4}$  7 groups of $\frac{3}{4}$  or $\frac{21}{4}$

Encourage students to use models or drawings to multiply or divide with fractions. Begin with students modeling multiplication and division with whole numbers. Have them explain how they used the model or drawing to arrive at the solution.

Models to consider when multiplying or dividing fractions include, but are not limited to: area models using rectangles or squares, fraction strips/bars and sets of counters.

Use calculators or models to explain what happens to the result of multiplying a whole number by a fraction ($3 \times \frac{1}{2}$, $4 \times \frac{1}{2}$, $5 \times \frac{1}{2}$ …and $4 \times \frac{1}{2}$, $4 \times \frac{1}{3}$, $4 \times \frac{1}{4}$,…) and when multiplying a fraction by a number greater than 1.

Use calculators or models to explain what happens to the result when dividing a unit fraction by a non-zero whole number ($\frac{1}{8} \div 4$, $\frac{1}{8} \div 8$, $\frac{1}{8} \div 16$,…) and what happens to the result when dividing a whole number by a unit fraction ($4 \div \frac{1}{4}$, $8 \div \frac{1}{4}$ $12 \div \frac{1}{4}$,…).

Present problem situations and have students use models and equations to solve the problem. It is important for students to develop understanding of multiplication and division of fractions through contextual situations.

<u>Common Misconceptions from Ohio's Model Curricula:</u>
Students may believe that multiplication always results in a larger number. Using models when multiplying with fractions will enable students to see that the results will be smaller.

Additionally, students may believe that division always results in a smaller number. Using models when dividing with fractions will enable students to see that the results will be larger.

- **I can add and subtract integers.**
- **I can multiply and divide integers.**

(From Ohio's Model Curricula) This cluster builds upon the understandings of rational numbers in Grade 6:

- quantities can be shown using + or – as having opposite directions or values,
- points on a number line show distance and direction,
- opposite signs of numbers indicate locations on opposite sides of 0 on the number line,
- the opposite of an opposite is the number itself,
- the absolute value of a rational number is its distance from 0 on the number line,
- the absolute value is the magnitude for a positive or negative quantity, and
- locating and comparing locations on a coordinate grid by using negative and positive numbers.

Learning now moves to exploring and ultimately formalizing rules for operations (addition, subtraction, multiplication and division) with integers.

Using both contextual and numerical problems, students should explore what happens when negatives and positives are combined. Number lines present a visual image for students to explore and record addition and subtraction results. Two-color counters or colored chips can be used as a physical and kinesthetic model for adding and subtracting integers. With one color designated to represent positives and a second color for negatives, addition/subtraction can be represented by placing the appropriate numbers of chips for the addends and their signs on a board. Using the notion of opposites, the board is simplified by removing pairs of opposite colored chips. The answer is the total of the remaining chips with the sign representing the appropriate color. Repeated opportunities over time will allow students to compare the results of adding and subtracting pairs of numbers, leading to the generalization of the rules. Fractional rational numbers and whole numbers should be used in computations and explorations.

Students should be able to give contextual examples of integer operations, write and solve equations for real-world problems and explain how the properties of operations apply. Real-world situations could include: profit/loss, money, weight, sea level, debit/credit, football yardage, etc.

Using what students already know about positive and negative whole numbers and multiplication with its relationship to division, students should generalize rules for multiplying and dividing rational numbers. Multiply or divide the same as for positive numbers, then designate the sign according to the number of negative factors. Students should analyze and solve problems leading to the generalization of the rules for operations with integers.

For example, beginning with known facts, students predict the answers for related facts, keeping in mind that the properties of operations apply (See Tables 1, 2 and 3 below).

Table 1	Table 2	Table 3
4 x 4 = 16	4 x 4 = 16	-4 x -4 = 16
4 x 3 = 12	4 x 3 = 12	-4 x -3 = 12
4 x 2 = 8	4 x 2 = 8	-4 x -2 = 8
4 x 1 = 4	4 x 1 = 4	-4 x -1 = 4
4 x 0 = 0	4 x 0 = 0	-4 x 0 = 0
4 x -1 =	-4 x 1 =	-1 x - 4 =
4 x - 2 =	-4 x 2 =	-2 x - 4 =
4 x - 3 =	-4 x 3 =	-3 x - 4 =
4 x - 4 =	-4 x 4 =	-4 x - 4 =

Using the language of “the opposite of” helps some students understand the multiplication of negatively signed numbers (-4 x -4 = 16, the opposite of 4 groups of -4). Discussion about the tables should address the patterns in the products, the role of the signs in the products and commutativity of multiplication. Then students should be asked to answer these questions and prove their responses.

- Is it always true that multiplying a negative factor by a positive factor results in a negative product?
- Does a positive factor times a positive factor always result in a positive product?
- What is the sign of the product of two negative factors?
- When three factors are multiplied, how is the sign of the product determined?
- How is the numerical value of the product of any two numbers found?

Students can use number lines with arrows and hops, groups of colored chips or logic to explain their reasoning. When using number lines, establishing which factor will represent the length, number and direction of the hops will facilitate understanding. Through discussion, generalization of the rules for multiplying integers would result.

Division of integers is best understood by relating division to multiplication and applying the rules. In time, students will transfer the rules to division situations. (Note: In 2b, this algebraic language ($-(p/q) = (-p)/q = p/(-q)$) is written for the teacher's information, not as an expectation for students.)

Ultimately, students should solve other mathematical and real-world problems requiring the application of these rules with fractions and decimals.

In Grade 7 the awareness of rational and irrational numbers is initiated by observing the result of changing fractions to decimals. Students should be provided with families of fractions, such as, sevenths, ninths, thirds, etc. to convert to decimals using long division. The equivalents can be grouped and named (terminating or repeating). Students should begin to see why these patterns occur. Knowing the formal vocabulary *rational* and *irrational* is not expected.

Strategies for Diverse Learners:
Problems of varying difficulty are incorporated into the problem sets.

Literacy Standards Considerations:

- **I can use order of operations.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can add and subtract rational numbers.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can multiply rational numbers.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can divide rational numbers.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can add and subtract integers.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can multiply and divide integers.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

Instructional Resources:

Glencoe McGraw Hill Algebra: Concepts and Applications © 2008 parts of Chapters 1, 2, 3, and 4

- **I can use order of operations.**

http://fawnnguyen.com/2012/10/02/foxy-fives---cuz-its-better-than-four-fours.aspx
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module1-2.html

- **I can add and subtract rational numbers.**

http://nrich.maths.org/5776
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module2-2.html

- **I can multiply rational numbers.**

http://mr-stadel.blogspot.com/2013/09/your-bf.html
http://nrich.maths.org/5776
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module2-3.html

- **I can divide rational numbers.**

http://mr-stadel.blogspot.com/2013/09/your-bf.html
http://nrich.maths.org/5776
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module2-4.html

- **I can add and subtract integers.**

http://www.teachesmath.com/?p=488
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module2-1.html

- **I can multiply and divide integers.**

http://www.teachesmath.com/?p=488

Assessment:
Formative:

- exit slips
- observation of students working on problems in class
- observation of student work from outside of class practice problems

Summative:

- unit test questions

Assessment Resources:
http://www.illustrativemathematics.org/
http://practice.parcc.testnav.com/
IIS
Glencoe McGraw Hill Algebra: Concepts and Applications © 2008

Unit 2 – Introduction to Algebra

(2 ½ weeks, September/October)

Common Core State Standards Addressed:

6-EE.2: Write, read, and evaluate expressions in which letters stand for numbers.
6-EE.3: Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y*.
6-EE.4: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*
A-CED.1: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.

Student Friendly Learning Targets (In order of teaching):

- I can translate words into algebraic expressions and equations.
- I can identify algebraic properties.
- I can evaluate algebraic expressions.
- I can simplify expressions.

Vocabulary:

expression, equation, Commutative Property of Addition, Commutative Property of Multiplication, Associative Property of Addition, Associative Property of Multiplication, Distributive Property, additive inverse, multiplicative inverse, Identity Property of Addition, Identity Property of Multiplication, Multiplication Property of Zero, terms, like terms, constant

Materials and/or Technology Needed:

calculator

Instructional Strategies:

Information from all Learning Targets can be found at Ohio's Model Curricula: http://education.ohio.gov/Topics/Academic-Content-Standards/Mathematics and KCTM's Flip Book (which encompasses several states' information including Ohio): http://www.katm.org/baker/pages/common-core-resources.php

- **I can translate words into algebraic expressions and equations.**

Provide examples of real-world problems that can be modeled by writing an equation or inequality. Begin with simple equations and inequalities and build up to more complex equations in two or more variables that may involve quadratic, exponential or rational functions.

- **I can identify algebraic properties.**
- **I can evaluate algebraic expressions.**
- **I can simplify expressions.**

(For all three from Ohio's Model Curricula) Provide opportunities for students to write expressions for numerical and real-world situations. Write multiple statements that represent a given algebraic expression. For example, the expression $x - 10$ could be written as "ten less than a number," "a number minus ten," "the temperature fell ten degrees,"', "I scored ten fewer points than my brother," etc. Students should also read an algebraic expression and write a statement.

Through modeling, encourage students to use proper mathematical vocabulary when discussing terms, factors, coefficients, etc.

Provide opportunities for students to write equivalent expressions, both numerically and with variables. For example, given the expression $x + x + x + x + 4•2$, students could write $2x + 2x + 8$ or some other equivalent expression. Make the connection to the simplest form of this expression as $4x + 8$. Because this is a foundational year for building the bridge between the concrete concepts of arithmetic and the abstract thinking of algebra, using hands-on materials (such as algebra tiles, counters, unifix cubes, "Hands on Algebra") to help students translate between concrete numerical representations and abstract symbolic representations is critical.

Provide expressions and formulas to students, along with values for the variables so students can evaluate the expression. Evaluate expressions using the order of operations with and without parentheses. Include whole-number exponents, fractions, decimals, etc. Provide a model that shows step-by-step thinking when simplifying an expression. This demonstrates how two lines of work maintain equivalent algebraic expressions and establishes the need to have a way to review and justify thinking.

Provide a variety of expressions and problem situations for students to practice and deepen their skills. Start with simple expressions to evaluate and move to more complex expressions. Likewise start with simple whole numbers and move to fractions and decimal numbers. The use of negatives and positives should mirror the level of introduction in Grade 6 The Number System; students are developing the concept and not generalizing operation rules.

The use of technology can assist in the exploration of the meaning of expressions. Many calculators will allow you to store a value for a variable and then use the variable in expressions. This enables the student to discover how the calculator deals with expressions like x^2, $5x$, xy, and $2(x + 5)$.

<u>Common Misconceptions from Ohio's Model Curricula:</u>

Many of the misconceptions when dealing with expressions stem from the misunderstanding/reading of the expression. For example, knowing the operations that are being referenced with notation like, x^3, $4x$, $3(x + 2y)$ is critical. The fact that x^3 means $x \bullet x \bullet x$, means x times x times x, not $3x$ or 3 times x; $4x$ means 4 times x or $x+x+x+x$, not forty-something. When evaluating $4x$ when $x = 7$, substitution does not result in the expression meaning 47. Use of the "*x*" notation as both the variable and the operation of multiplication can complicate this understanding.

Strategies for Diverse Learners:

Problems of varying difficulty are incorporated into the problem sets.

Literacy Standards Considerations:

- **I can translate words into algebraic expressions and equations.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can identify algebraic properties.**

Students would need to be familiar with the definitions of key terms.

- **I can evaluate algebraic expressions.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can simplify expressions.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

Instructional Resources:

Glencoe McGraw Hill Algebra: Concepts and Applications © 2008 parts of Chapter 1

- **I can translate words into algebraic expressions and equations.**

http://map.mathshell.org/materials/lessons.php?taskid=221&subpage=concept
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module1-1.html

- **I can identify algebraic properties.**

http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module1-4.html
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module1-5.html
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module1-6.html

- **I can evaluate algebraic expressions.**

http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module1-3.html

- **I can simplify expressions.**

Assessment:

Formative:

- exit slips
- observation of students working on problems in class
- observation of student work from outside of class practice problems

Summative:

- unit test questions

Assessment Resources:

http://www.illustrativemathematics.org/
http://practice.parcc.testnav.com/
IIS
Glencoe McGraw Hill Algebra: Concepts and Applications © 2008

Unit 3 – Solving Linear Equations

(4 weeks, October/November)

Common Core State Standards Addressed (In order of teaching):

6-EE.5: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
A-CED.1: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.
A-CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*.
A-REI.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Student Friendly Learning Targets:

- I can determine whether a given number is a solution of an equation.
- I can solve addition and subtraction equations.
- I can solve multiplication and division equations.
- I can solve equations involving more than one operation.
- I can solve equations with variables on both sides.
- I can equations with grouping symbols.

Vocabulary:

linear, linear equation, coefficient, solutions

Materials and/or Technology Needed:

calculator

Instructional Strategies:

Information from all Learning Targets can be found at Ohio's Model Curricula: http://education.ohio.gov/Topics/Academic-Content-Standards/Mathematics and KCTM's Flip Book (which encompasses several states' information including Ohio): http://www.katm.org/baker/pages/common-core-resources.php

- **I can determine whether a given number is a solution of an equation.**

(From Ohio's Model Curricula) The skill of solving an equation must be developed *conceptually* before it is developed *procedurally*. This means that students should be thinking about what numbers could possibly be a solution to the equation before solving the equation. For example, in the equation $x + 21 = 32$ students know that $21 + 9 = 30$ therefore the solution must be 2 more than 9 or 11, so $x = 11$.

Provide multiple situations in which students must determine if a single value is required as a solution, or if the situation allows for multiple solutions. This creates the need for both types of equations (single solution for the situation) and inequalities (multiple solutions for the situation). Solutions to equations should not require using the rules for operations with negative numbers since the conceptual understanding of negatives and positives is being introduced in Grade 6. When working with inequalities, provide situations in which the solution is not limited to the set of positive whole numbers but includes rational numbers. This is a good way to practice fractional numbers and introduce negative numbers. Students need to be aware that numbers less than zero could be part of a solution set for a situation. As an extension to this concept, certain situations may require a solution between two numbers. For example, a problem situation may have a solution that requires more than 10 but not greater than 25. Therefore, the exploration with students as to what this would look like both on a number line and symbolically is a reasonable extension.

The process of translating between mathematical phrases and symbolic notation will also assist students in the writing of equations/inequalities for a situation. This process should go both ways; Students should be able to write a mathematical phrase for an equation. Additionally, the writing of equations from a situation or story does not come naturally for many students. A strategy for assisting with this is to give students an equation and ask them to come up with the situation/story that the equation could be referencing.

- **I can solve addition and subtraction equations.**
- **I can solve multiplication and division equations.**
- **I can solve equations involving more than one operation.**
- **I can solve equations with variables on both sides.**
- **I can equations with grouping symbols.**

(From Ohio's Model Curricula) Provide examples of real-world problems that can be modeled by writing an equation or inequality. Begin with simple equations and inequalities and build up to more complex equations in two or more variables that may involve quadratic, exponential or rational functions.

Provide examples of real-world problems that can be solved by writing an equation, and have students explore the graphs of the equations on a graphing calculator to determine which parts of the graph are relevant to the problem context.

Challenge students to justify each step of solving an equation. Transforming $2x - 5 = 7$ to $2x = 12$ is possible because 5 = 5, so adding the same quantity to both sides of an equation makes the resulting equation true as well. Each step of solving an equation can be defended, much like providing evidence for steps of a geometric proof.

Provide examples for how the same equation might be solved in a variety of ways as long as equivalent quantities are added or subtracted to both sides of the equation, the order of steps taken will not matter.

$3n + 2 = n - 10$	$3n + 2 = n - 10$	$3n + 2 = n - 10$
$\underline{-\quad 2 = \quad -2}$	$\underline{+\quad 10 = \quad +10}$	$\underline{-n \quad = -n}$
$3n \quad = n - 12$ OR	$3n + 12 = n$ OR	$2n + 2 = -10$
$\underline{-n \quad = -n}$	$\underline{-3n \quad = -3n}$	$\underline{\quad -2 = -2}$
$2n \quad = -12$	$12 = -2n$	$2n = -12$
$n = -6$	$n = -6$	$n = -6$

Begin with simple, one-step equations and require students to write out a justification for each step used to solve the equation.

There are two major reasons for discussing the topic of inequalities along with equations: First, there are analogies between solving equations and inequalities that help students understand them both**.** Second, the applications that lead to equations almost always lead in the same way to inequalities.

In grades 6-8, students solve and graph linear equations and inequalities. It may be beneficial to remind students of the most common solving techniques, such as converting fractions from one form to another, removing parentheses in the sentences, or multiplying both sides of an equation or inequality by the common denominator of the fractions.

Careful selection of examples and exercises is needed to provide students with meaningful review and to introduce other important concepts, such as the use of properties and applications of solving linear equations and inequalities. Stress the idea that the application of properties is also appropriate when working with equations or inequalities that include more than one variable, fractions and decimals. Regardless of the type of numbers or variables in the equation or inequality, students have to examine the validity of each step in the solution process.

<u>Common Misconceptions from Ohio's Model Curricula:</u>
Students may believe that equations of linear, quadratic and other functions are abstract and exist only "in a math book," without seeing the usefulness of these functions as modeling real-world phenomena.

Students may believe that solving an equation such as $3x + 1 = 7$ involves "only removing the 1," failing to realize that the equation 1 = 1 is being subtracted to produce the next step.

Some students may believe that for equations containing fractions only on one side, it requires "clearing fractions" (the use of multiplication) only on that side of the equation. To address this misconception, start by demonstrating the solution methods for equations similar to $\frac{1}{4}x + \frac{1}{5}x +$

$\frac{1}{6}$x + 46 = x and stress that the Multiplication Property of Equality is applied to both sides, which are multiplied by 60.

Strategies for Diverse Learners:
Problems of varying difficulty are incorporated into the problem sets.

Literacy Standards Considerations:
- **I can determine whether a given number is a solution of an equation.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can solve addition and subtraction equations.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can solve multiplication and division equations.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can solve equations involving more than one operation.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can solve equations with variables on both sides.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can equations with grouping symbols.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

Instructional Resources:
Glencoe McGraw Hill Algebra: Concepts and Applications © 2008 parts of Chapters 3 and 4
- **I can determine whether a given number is a solution of an equation.**

- **I can solve addition and subtraction equations.**

http://simplifyingradicals2.blogspot.com/2013/10/sticky-clouds.html
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module3-1.html

- **I can solve multiplication and division equations.**

http://simplifyingradicals2.blogspot.com/2013/10/sticky-clouds.html
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module3-2.html

- **I can solve equations involving more than one operation.**

http://highschoolmathfun.blogspot.com/2014/02/multi-step-equations.html
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module3-3.html

- **I can solve equations with variables on both sides.**

http://map.mathshell.org/materials/lessons.php?taskid=554&subpage=concept
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module3-4.html

- **I can equations with grouping symbols.**

http://exponentialcurve.blogspot.com/2013/10/constructing-and-deconstructing.html
http://map.mathshell.org/materials/lessons.php?taskid=487&subpage=concept
http://justtellmetheanswer.wordpress.com/2013/08/30/using-common-core-you-need-desmos/ (first topic)
http://everybodyisageniusblog.blogspot.com/2012/07/solving-special-case-equations.html
http://simplifyingradicals2.blogspot.com/2013/10/buttons-and-applications-of-linear.html
http://simplifyingradicals2.blogspot.com/2013/10/snowflake-posters-and-multi-step.html

Assessment:
Formative:

- exit slips
- observation of students working on problems in class
- observation of student work from outside of class practice problems

Summative:

- unit test questions

Assessment Resources:
http://www.illustrativemathematics.org/
http://practice.parcc.testnav.com/
IIS
Glencoe McGraw Hill Algebra: Concepts and Applications © 2008

Unit 4 – Linear Inequalities

(3 ½ weeks, November/December)

Common Core State Standards Addressed (In order of teaching):

6-EE.8: Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
A-CED.1: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.
A-REI.1: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
A-REI.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Student Friendly Learning Targets:

- I can graph linear inequalities on a number line.
- I can solve linear inequalities involving one operation.
- I can solve linear inequalities involving more than one operation.
- I can solve and graph a compound inequality in one variable.
- I can solve an absolute value equation in one variable.

Vocabulary:

linear, linear equation, linear inequality, coefficient, constraints, solutions, formula, literal equation, absolute value, compound inequality, and, or, intersection, union, half-plane, boundary, coordinate plane

Materials and/or Technology Needed:

calculator, graphing calculator, Desmos.com, graph paper, ruler

Instructional Strategies:

Information from all Learning Targets can be found at Ohio's Model Curricula: http://education.ohio.gov/Topics/Academic-Content-Standards/Mathematics and KCTM's Flip Book (which encompasses several states' information including Ohio): http://www.katm.org/baker/pages/common-core-resources.php

- **I can graph linear inequalities on a number line.**

(From Ohio's Model Curricula) Provide multiple situations in which students must determine if a single value is required as a solution, or if the situation allows for multiple solutions. This creates the need for both types of equations (single solution for the situation) and inequalities (multiple solutions for the situation). Solutions to equations should not require using the rules for operations with negative numbers since the conceptual understanding of negatives and positives is being introduced in Grade 6. When working with inequalities, provide situations in which the solution is not limited to the set of positive whole numbers but includes rational numbers. This is a good way to practice fractional numbers and introduce negative numbers. Students need to be aware that numbers less than zero could be part of a solution set for a situation. As an extension to this concept, certain situations may require a solution between two numbers. For example, a problem situation may have a solution that requires more than 10 but not greater than 25. Therefore, the exploration with students as to what this would look like both on a number line and symbolically is a reasonable extension.

- **I can solve linear inequalities involving one operation.**
- **I can solve linear inequalities involving more than one operation.**

(From Ohio's Model Curricula) Provide examples of real-world problems that can be modeled by writing an equation or inequality. Begin with simple equations and inequalities and build up to more complex equations in two or more variables that may involve quadratic, exponential or rational functions.

There are two major reasons for discussing the topic of inequalities along with equations: First, there are analogies between solving equations and inequalities that help students understand them both. Second, the applications that lead to equations almost always lead in the same way to inequalities.

In grades 6-8, students solve and graph linear equations and inequalities. Graphing experience with inequalities is limited to graphing on a number line diagram. Despite this work, some students will still need more practice to be proficient. It may be beneficial to remind students of the most common solving techniques, such as converting fractions from one form to another, removing parentheses in the sentences, or multiplying both sides of an equation or inequality by the common denominator of the fractions. Students must be aware of what it means to check an inequality's solution. The substitution of the end points of the solution set in the original inequality should give equality regardless of the presence or the absence of an equal sign in the original sentence. The substitution of any value from the rest of the solution set should give a correct inequality.

Careful selection of examples and exercises is needed to provide students with meaningful review and to introduce other important concepts, such as the use of properties and applications of solving linear equations and inequalities. Stress the idea that the application of properties is also appropriate when working with equations or inequalities that include more than one variable, fractions and decimals. Regardless of the type of numbers or variables in the equation or inequality, students have to examine the validity of each step in the solution process.

Common Misconceptions from Ohio's Model Curricula:

Students may confuse the rule of changing a sign of an inequality when multiplying or dividing by a negative number with changing the sign of an inequality when one or two sides of the inequality become negative (for ex., $3x > -15$ or $x < -5$).

- **I can solve and graph a compound inequality in one variable.**

(From Ohio's Model Curricula) There are two major reasons for discussing the topic of inequalities along with equations: First, there are analogies between solving equations and inequalities that help students understand them both. Second, the applications that lead to equations almost always lead in the same way to inequalities.

In grades 6-8, students solve and graph linear equations and inequalities. Graphing experience with inequalities is limited to graphing on a number line diagram. Despite this work, some students will still need more practice to be proficient. It may be beneficial to remind students of the most common solving techniques, such as converting fractions from one form to another, removing parentheses in the sentences, or multiplying both sides of an equation or inequality by the common denominator of the fractions. Students must be aware of what it means to check an inequality's solution. The substitution of the end points of the solution set in the original inequality should give equality regardless of the presence or the absence of an equal sign in the original sentence. The substitution of any value from the rest of the solution set should give a correct inequality.

Careful selection of examples and exercises is needed to provide students with meaningful review and to introduce other important concepts, such as the use of properties and applications of solving linear equations and inequalities. Stress the idea that the application of properties is also appropriate when working with equations or inequalities that include more than one variable, fractions and decimals. Regardless of the type of numbers or variables in the equation or inequality, students have to examine the validity of each step in the solution process.

<u>Common Misconceptions from Ohio's Model Curricula:</u>
Students may confuse the rule of changing a sign of an inequality when multiplying or dividing by a negative number with changing the sign of an inequality when one or two sides of the inequality become negative (for ex., $3x > -15$ or $x < -5$).

- **I can solve an absolute value equation in one variable.**

(From Ohio's Model Curricula) Challenge students to justify each step of solving an equation. Transforming $2x - 5 = 7$ to $2x = 12$ is possible because 5 = 5, so adding the same quantity to both sides of an equation makes the resulting equation true as well. Each step of solving an equation can be defended, much like providing evidence for steps of a geometric proof.

Provide examples for how the same equation might be solved in a variety of ways as long as equivalent quantities are added or subtracted to both sides of the equation, the order of steps taken will not matter.

$3n + 2 = n - 10$		$3n + 2 = n - 10$		$3n + 2 = n - 10$
$-\quad 2 = \quad -2$		$+\quad 10 = \quad +10$		$-n \quad = -n$
$3n \quad = n - 12$	OR	$3n + 12 = n$	OR	$2n + 2 = -10$
$-n \quad = -n$		$-3n \quad = -3n$		$-2 = -2$

$2n = -12$	$12 = -2n$	$2n = -12$
$n = -6$	$n = -6$	$n = -6$

Begin with simple, one-step equations and require students to write out a justification for each step used to solve the equation.

There are two major reasons for discussing the topic of inequalities along with equations: First, there are analogies between solving equations and inequalities that help students understand them both. Second, the applications that lead to equations almost always lead in the same way to inequalities.

In grades 6-8, students solve and graph linear equations and inequalities. Graphing experience with inequalities is limited to graphing on a number line diagram. Despite this work, some students will still need more practice to be proficient. It may be beneficial to remind students of the most common solving techniques, such as converting fractions from one form to another, removing parentheses in the sentences, or multiplying both sides of an equation or inequality by the common denominator of the fractions.

Careful selection of examples and exercises is needed to provide students with meaningful review and to introduce other important concepts, such as the use of properties and applications of solving linear equations and inequalities. Stress the idea that the application of properties is also appropriate when working with equations or inequalities that include more than one variable, fractions and decimals. Regardless of the type of numbers or variables in the equation or inequality, students have to examine the validity of each step in the solution process.

<u>Common Misconceptions from Ohio's Model Curricula:</u>
Students may believe that solving an equation such as $3x + 1 = 7$ involves "only removing the 1," failing to realize that the equation 1 = 1 is being subtracted to produce the next step.

- **I can solve an absolute value inequality in one variable.**

(From Ohio's Model Curricula) There are two major reasons for discussing the topic of inequalities along with equations: First, there are analogies between solving equations and inequalities that help students understand them both. Second, the applications that lead to equations almost always lead in the same way to inequalities.

In grades 6-8, students solve and graph linear equations and inequalities. Graphing experience with inequalities is limited to graphing on a number line diagram. Despite this work, some students will still need more practice to be proficient. It may be beneficial to remind students of the most common solving techniques, such as converting fractions from one form to another, removing parentheses in the sentences, or multiplying both sides of an equation or inequality by the common denominator of the fractions. Students must be aware of what it means to check an inequality's solution. The substitution of the end points of the solution set in the original inequality should give equality regardless of the presence or the absence of an equal sign in the original sentence**.** The substitution of any value from the rest of the solution set should give a correct inequality.

Careful selection of examples and exercises is needed to provide students with meaningful review and to introduce other important concepts, such as the use of properties and applications of solving linear equations and inequalities. Stress the idea that the application of properties is also appropriate when working with equations or inequalities that include more than one variable, fractions and decimals. Regardless of the type of numbers or variables in the equation or inequality, students have to examine the validity of each step in the solution process.

Common Misconceptions from Ohio's Model Curricula:
Students may confuse the rule of changing a sign of an inequality when multiplying or dividing by a negative number with changing the sign of an inequality when one or two sides of the inequality become negative (for ex., $3x > -15$ or $x < -5$).

Strategies for Diverse Learners:
Problems of varying difficulty are incorporated into the problem sets.

Literacy Standards Considerations:

- **I can graph linear inequalities on a number line.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can solve linear inequalities involving one operation.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can solve linear inequalities involving more than one operation.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can solve and graph a compound inequality in one variable.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can solve an absolute value equation in one variable.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can solve an absolute value inequality in one variable.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

Instructional Resources:
Glencoe McGraw Hill Algebra: Concepts and Applications © 2008 parts of Chapters 3 and 12

- **I can graph linear inequalities on a number line.**

- **I can solve linear inequalities involving one operation.**

http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module4-1.html
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module4-2.html

- **I can solve linear inequalities involving more than one operation.**

http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module4-3.html

- **I can solve and graph a compound inequality in one variable.**

http://highschoolmathfun.blogspot.com/2014/03/compound-inequalities.html
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module4-4.html

- **I can solve an absolute value equation in one variable.**

http://function-of-time.blogspot.com/2011/09/algebra-2-solving-absolute-value.html
http://clopendebate.wordpress.com/2013/10/16/teaching-absolute-value-equationsinequalities/

- **I can solve an absolute value inequality in one variable.**

http://clopendebate.wordpress.com/2013/10/16/teaching-absolute-value-equationsinequalities/

Assessment:
Formative:

- exit slips
- observation of students working on problems in class
- observation of student work from outside of class practice problems

Summative:

- unit test questions

Assessment Resources:
http://www.illustrativemathematics.org/
http://practice.parcc.testnav.com/
IIS
Glencoe McGraw Hill Algebra: Concepts and Applications © 2008

Unit 5 – Representing Relationships Mathematically

(3 weeks, January)

<u>Common Core State Standards Addressed:</u>

N-Q.2: Define appropriate quantities for the purpose of descriptive modeling.
A-SSE.1a: Interpret parts of an expression, such as terms, factors, and coefficients.
A-CED.1: Create equations and inequalities in one variable and use them to solve problems.
A-CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
A-CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
F-IF.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*[*]
F-BF.1a: Write a function that describes a relationship between two quantities.[*]

<u>Student Friendly Learning Targets (In order of teaching):</u>

- I can identify parts of an expression.
- I can create equations in two variables to represent relationships between quantities.
- I can interpret solutions in the context of the situation modeled and decide if they are reasonable.
- I can determine the domain for a given function in two variables.

<u>Vocabulary:</u>

expression, equation, term, variable, coefficient, constant, like terms, degree, inequality, viable, non-viable, ordered pair, coordinate plane, axes, coordinate, function, domain, range, x-intercept, y-intercept

<u>Materials and/or Technology Needed:</u>

calculator, graphing calculator, Desmos.com, graph paper, ruler

<u>Instructional Strategies:</u>

Information from all Learning Targets can be found at Ohio's Model Curricula: http://education.ohio.gov/Topics/Academic-Content-Standards/Mathematics and KCTM's Flip Book (which encompasses several states' information including Ohio): http://www.katm.org/baker/pages/common-core-resources.php

- **I can identify parts of an expression.**

(From KCTM Flip Book) Extending beyond simplifying an expression, this cluster addresses interpretation of the components in an algebraic expression. A student should recognize that in the expression 2x + 1, "2" is the coefficient, "2" and "x" are factors, and "1" is a constant, as well as “2x” and “1” being terms of the binomial expression. Development and proper use of mathematical language is an important building block for future content. Using real-world context examples, the nature of algebraic expressions can be explored.

Common Misconceptions (from KCTM Flip Book):
Students may believe that the use of algebraic expressions is merely the abstract manipulation of symbols. Use of real- world context examples to demonstrate the meaning of the parts of algebraic expressions is needed to counter this misconception.

Students will often combine terms that are not like terms. For example, $2 + 3x = 5x$ or $3x + 2y = 5xy$.

Students sometimes forget the coefficient of 1 when adding like terms. For example, $x + 2x + 3x = 5x$ rather than $6x$.

Students will change the degree of the variable when adding/subtracting like terms. For example, $2x + 3x = 5x^2$ rather than $5x$.

Students will forget to distribute to all terms when multiplying. For example, $6(2x + 1) = 12x + 1$ rather than $12x + 6$.

Students may not follow the Order of Operations when simplifying expressions. For example, $4x^2$ when $x = 3$ may be incorrectly evaluated as $4 \bullet 32 = 12^2 = 144$, rather than $4 \bullet 9 = 36$. Another common mistake occurs when the distributive property should be used prior to adding/subtracting. For example, $2 + 3(x – 1)$ incorrectly becomes $5(x – 1) = 5x – 5$ instead of $2 + 3(x – 1) = 2 + 3x – 3 = 3x – 1$.

Students fail to use the property of exponents correctly when using the distributive property. For example, $3x(2x – 1) = 6x – 3x = 3x$ instead of simplifying as $3x(2x – 1) = 6x^2 – 3x$.

Students fail to understand the structure of expressions. For example, they will write $4x$ when $x = 3$ is 43 instead of $4x = 4 \bullet x$ so when $x = 3$, $4x = 4 \bullet 3 = 12$. In addition, students commonly misevaluate $–3^2 = 9$ rather than $–3^2 = –9$. Students routinely see $–3^2$ as the same as $(–3)^2 = 9$. A method that may clear up the misconception is to have students rewrite as $–x^2 = –1 \bullet x^2$ so they know to apply the exponent before the multiplication of –1.

Students frequently attempt to “solve” expressions. Many students add “= 0” to an expression they are asked to simplify. Students need to understand the difference between an equation and an expression.

Students commonly confuse the properties of exponents, specifically the product of powers property with the power of a power property. For example, students will often simplify $(x^2)^3 = x^5$ instead of x^6.

Students will incorrectly translate expressions that contain a difference of terms. For example, 8 less than 5 times a number is often incorrectly translated as 8 – 5n rather than 5n – 8.

- **I can create equations in two variables to represent relationships between quantities.**

(From Ohio's Model Curricula) Using real-world context examples, the nature of algebraic expressions can be explored**.** For example, suppose the cost of cell phone service for a month is represented by the expression 0.40*s* + 12.95**.** Students can analyze how the coefficient of 0.40 represents the cost of one minute (40¢), while the constant of 12.95 represents a fixed, monthly fee, and *s* stands for the number of cell phone minutes used in the month. Similar real-world examples, such as tax rates, can also be used to explore the meaning of expressions.

Provide examples of real-world problems that can be modeled by writing an equation or inequality. Begin with simple equations and inequalities and build up to more complex equations in two or more variables that may involve quadratic, exponential or rational functions.

Explore examples illustrating when it is useful to rewrite a formula by solving for one of the variables in the formula. For example, the formula for the area of a trapezoid ($A = \frac{1}{2}h(b_1 + b_2)$) can be solved for *h* if the area and lengths of the bases are known but the height needs to be calculated. This strategy of selecting a different representation has many applications in science and business when using formulas.

Provide examples of real-world problems that can be solved by writing an equation, and have students explore the graphs of the equations on a graphing calculator to determine which parts of the graph are relevant to the problem context.

Give students formulas, such as area and volume (or from science or business), and have students solve the equations for each of the different variables in the formula.

Provide a real-world example (e.g., a table showing how far a car has driven after a given number of minutes, traveling at a uniform speed), and examine the table by looking "down" the table to describe a recursive relationship, as well as "across" the table to determine an explicit formula to find the distance traveled if the number of minutes is known.

Write out terms in a table in an expanded form to help students see what is happening. For example, if the *y*-values are 2, 4, 8, 16, they could be written as 2, 2(2), 2(2)(2), 2(2)(2)(2), etc., so that students recognize that 2 is being used multiple times as a factor.

Focus on one representation and its related language – recursive or explicit – at a time so that students are not confusing the formats.

Common Misconceptions (from Ohio's Model Curricula):

Students may believe that equations of linear, quadratic and other functions are abstract and exist only “in a math book,” without seeing the usefulness of these functions as modeling real-world phenomena.

(From KCTM Flip Book) Students may interchange slope and y-intercept when creating equations. For example, a taxi cab costs $4 for a dropped flag and charges $2 per mile. Students may fail to see that $2 is a rate of change and is slope while the $4 is the starting cost and incorrectly write the equation as $y = 4x + 2$ instead of $y = 2x + 4$.

Students do not correctly identify whether a situation should be represented by a linear, quadratic, or exponential function.

Students often do not understand what the variables represent. For example, if the height h in feet of a piece of lava t seconds after it is ejected from a volcano is given by $h(t) = -16t^2 + 64t + 936$ and the student is asked to find the time it takes for the piece of lava to hit the ground, the student will have difficulties understanding that $h = 0$ at the ground and that they need to solve for t.

- **I can interpret solutions in the context of the situation modeled and decide if they are reasonable.**

(From Ohio’s Model Curricula) Examine real-world graphs in terms of constraints that are necessary to balance a mathematical model with the real-world context. For example, a student writing an equation to model the maximum area when the perimeter of a rectangle is 12 inches should recognize that $y = x(6 – x)$ only makes sense when $0 < x < 6$. This restriction on the domain is necessary because the side of a rectangle under these conditions cannot be less than or equal to 0, but must be less than 6. Students can discuss the difference between the parabola that models the problem and the portion of the parabola that applies to the context.

Common Misconceptions (from Ohio’s Model Curricula):
Students may believe that equations of linear, quadratic and other functions are abstract and exist only “in a math book,” without seeing the usefulness of these functions as modeling real-world phenomena.

- **I can determine the domain for a given function in two variables.**

(From Ohio’s Model Curricula) Recognize appropriate domains of functions in real-world settings. For example, when determining a weekly salary based on hours worked, the hours (input) could be a rational number, such as 25.5. However, if a function relates the number of cans of soda sold in a machine to the money generated, the domain must consist of whole numbers

Common Misconceptions (from Ohio’s Model Curricula):
Students may believe that it is reasonable to input any *x*-value into a function, so they will need to examine multiple situations in which there are various limitations to the domains.

<u>Strategies for Diverse Learners:</u>
Problems of varying difficulty are incorporated into the problem sets.

<u>Literacy Standards Considerations:</u>

- **I can identify parts of an expression.**

Students would need to be familiar with the definitions of key terms.

- **I can create equations in two variables to represent relationships between quantities.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can interpret solutions in the context of the situation modeled and decide if they are reasonable.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can determine the domain for a given function in two variables.**

Students will need to interpret the problem situation to determine what answers make sense.

<u>Instructional Resources:</u>
Glencoe McGraw Hill Algebra: Concepts and Applications © 2008 parts of Chapter 6

- **I can identify parts of an expression.**

http://map.mathshell.org/materials/lessons.php?taskid=221&subpage=concept

- **I can create equations in two variables to represent relationships between quantities.**

http://everybodyisageniusblog.blogspot.com/2012/09/translating-math.html
http://www.insidemathematics.org/problems-of-the-month/pom-growingstaircases.pdf
http://fawnnguyen.com/2013/02/05/20130204.aspx
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module1-1.html

- **I can interpret solutions in the context of the situation modeled and decide if they are reasonable.**

http://www.insidemathematics.org/common-core-math-tasks/high-school/HS-A-2003%20Number%20Towers.pdf
http://www.insidemathematics.org/problems-of-the-month/pom-surroundedandcovered.pdf
http://www.insidemathematics.org/problems-of-the-month/pom-perfectpair.pdf
Many of the resources for writing the equation or inequality that best models the problem also will work here.

- **I can determine the domain for a given function in two variables.**

http://www.illustrativemathematics.org/illustrations/387
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module5-2.html

Assessment:

Formative:

- exit slips
- observation of students working on problems in class
- observation of student work from outside of class practice problems

Summative:

- unit test questions

Assessment Resources:

http://www.illustrativemathematics.org/
http://practice.parcc.testnav.com/
IIS
Glencoe McGraw Hill Algebra: Concepts and Applications © 2008

Unit 6 – Understanding Functions

(4 weeks, January/February)

<u>Common Core State Standards Addressed:</u>

N-Q.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

A-REI.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

F-IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

F-IF.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

F-IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.*

F-IF.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

F-IF.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum*.

<u>Student Friendly Learning Targets (In order of teaching):</u>

- I can determine if a relation is a function.
- I can evaluate functions using function notation.
- I can convert a list of numbers (a sequence) into a function.
- I can graph an equation on the coordinate plane using a table.
- I can compare properties of two functions graphically, in table form, and algebraically.
- I can find key features of a graph using a graph, a table, or an equation.
- I can state the appropriate domain of a function that represents a problem situation.

<u>Vocabulary:</u>

function, domain, range, input, output, function notation, arithmetic sequence, maximum, minimum, end behavior increasing, decreasing

Materials and/or Technology Needed:
calculator, graphing calculator, Desmos.com, graph paper, ruler

Instructional Strategies:
Information from all Learning Targets can be found at Ohio's Model Curricula: http://education.ohio.gov/Topics/Academic-Content-Standards/Mathematics and KCTM's Flip Book (which encompasses several states' information including Ohio): http://www.katm.org/baker/pages/common-core-resources.php

- **I can determine if a relation is a function.**

(From Ohio's Model Curricula) Provide applied contexts in which to explore functions. For example, examine the amount of money earned when given the number of hours worked on a job, and contrast this with a situation in which a single fee is paid by the "carload" of people, regardless of whether 1, 2, or more people are in the car.

Help students to understand that the word "domain" implies the set of all possible input values and that the integers are a set of numbers made up of {…-2, -1, 0, 1, 2, …}.

Distinguish between relationships that are not functions and those that are functions (e.g., present a table in which one of the input values results in multiple outputs to contrast with a functional relationship). Examine graphs of functions and non-functions, recognizing that if a vertical line passes through at least two points in the graph, then y (or the quantity on the vertical axis) is not a function of x (or the quantity on the horizontal axis).

Common Misconceptions (from Ohio's Model Curricula):
Students may believe that all relationships having an input and an output are functions, and therefore, misuse the function terminology.

- **I can evaluate functions using function notation.**

(From Ohio's Model Curricula) Provide applied contexts in which to explore functions. For example, examine the amount of money earned when given the number of hours worked on a job, and contrast this with a situation in which a single fee is paid by the "carload" of people, regardless of whether 1, 2, or more people are in the car.

Use diagrams to help students visualize the idea of a function machine. Students can examine several pairs of input and output values and try to determine a simple rule for the function.

Common Misconceptions (from Ohio's Model Curricula):
Students may also believe that the notation $f(x)$ means to multiply some value f times another value x. The notation alone can be confusing and needs careful development. For example, $f(2)$ means the output value of the function f when the input value is 2.

- **I can convert a list of numbers (a sequence) into a function.**

(From Ohio's Model Curricula) Rewrite sequences of numbers in tabular form, where the input represents the term number (the position or index) in the sequence, and the output represents the number in the sequence.

- **I can graph an equation on the coordinate plane using a table.**

(From Ohio's Model Curricula) Discuss the importance of using appropriate labels and scales on the axes when representing functions with graphs.

Use a graphing calculator or Desmos.com to demonstrate how dramatically the shape of a curve can change when the scale of the graph is altered for one or both variables.

Beginning with simple, real-world examples, help students to recognize a graph as a set of solutions to an equation. For example, if the equation $y = 6x + 5$ represents the amount of money paid to a babysitter (i.e., $5 for gas to drive to the job and $6/hour to do the work), then every point on the line represents an amount of money paid, given the amount of time worked.

Using technology, have students graph a function and use the trace function to move the cursor along the curve. Discuss the meaning of the ordered pairs that appear at the bottom of the calculator, emphasizing that every point on the curve represents a solution to the equation. Begin with simple linear equations and how to solve them using the graphs and tables on a graphing calculator. Then, advance students to nonlinear situations so they can see that even complex equations that might involve quadratics, absolute value, or rational functions can be solved fairly easily using this same strategy. While a standard graphing calculator does not simply solve an equation for the user, it can be used as a tool to approximate solutions.

Common Misconceptions (from Ohio's Model Curricula):
Students believe that the labels and scales on a graph are not important and can be assumed by a reader, and that it is always necessary to use the entire graph of a function when solving a problem that uses that function as its model.

Students may believe that the graph of a function is simply a line or curve "connecting the dots," without recognizing that the graph represents all solutions to the equation.

Additionally, students may believe that two-variable inequalities have no application in the real world.

- **I can compare properties of two functions graphically, in table form, and algebraically.**

(From Ohio's Model Curricula) Graphical representations serve as visual models for understanding phenomena that take place in our daily surroundings. The use of different kinds of graphical representations along with their units, labels and titles demonstrate the level of students' understanding and foster the ability to reason, prove, self-check, examine relationships and establish the validity of arguments. Students need to be able to identify misleading graphs by choosing correct units and scales to create a correct representation of a situation or to make a correct conclusion from it.

Discuss the importance of using appropriate labels and scales on the axes when representing functions with graphs.

Use a graphing calculator or Desmos.com to demonstrate how dramatically the shape of a curve can change when the scale of the graph is altered for one or both variables.

<u>Common Misconceptions (from Ohio's Model Curricula):</u>
Students believe that the labels and scales on a graph are not important and can be assumed by a reader, and that it is always necessary to use the entire graph of a function when solving a problem that uses that function as its model.

(From KCTM Flip Book) Given a graph of a line, students use the x-intercept for b instead of the y-intercept.

Given a graph, students incorrectly compute slope as run over rise rather than rise over run. For example, they will compute slope with the change in x over the change in y.

- **I can find key features of a graph using a graph, a table, or an equation.**

(From Ohio's Model Curricula) Flexibly move from examining a graph and describing its characteristics (e.g., intercepts, relative maximums, etc.) to using a set of given characteristics to sketch the graph of a function.

Examine a table of related quantities and identify features in the table, such as intervals on which the function increases, decreases, or exhibits periodic behavior.

Begin with simple, linear functions to describe features and representations, and then move to more advanced functions, including non-linear situations.

Provide students with many examples of functional relationships, both linear and non-linear. Use real-world examples, such as the growth of an investment fund over time, so that students can not only describe what they see in a table, equation, or graph, but also can relate the features to the real-life meanings.

Allow students to collect their own data sets, such as the falling temperature of a glass of hot water when removed from a flame versus the amount of time, to generate tables and graphs for discussion and interpretation.

- **I can state the appropriate domain of a function that represents a problem situation.**

(From Ohio's Model Curricula) Recognize appropriate domains of functions in real-world settings. For example, when determining a weekly salary based on hours worked, the hours (input) could be a rational number, such as 25.5. However, if a function relates the number of cans of soda sold in a machine to the money generated, the domain must consist of whole numbers.

Begin with simple, linear functions to describe features and representations, and then move to more advanced functions, including non-linear situations.

Provide students with many examples of functional relationships, both linear and non-linear. Use real-world examples, such as the growth of an investment fund over time, so that students can not only describe what they see in a table, equation, or graph, but also can relate the features to the real-life meanings.

Allow students to collect their own data sets, such as the falling temperature of a glass of hot water when removed from a flame versus the amount of time, to generate tables and graphs for discussion and interpretation.

Common Misconceptions (from Ohio's Model Curricula):
Students may believe that it is reasonable to input any x-value into a function, so they will need to examine multiple situations in which there are various limitations to the domains.

Strategies for Diverse Learners:
Problems of varying difficulty are incorporated into the problem sets.

Literacy Standards Considerations:

- **I can determine if a relation is a function.**

Students would need to be familiar with the definitions of key terms.

- **I can evaluate functions using function notation.**

Students would need to be familiar with the definitions of key terms.

- **I can convert a list of numbers (a sequence) into a function.**

Students would need to be familiar with the definitions of key terms.

- **I can graph an equation on the coordinate plane using a table.**

Students will need to interpret the problem situation to determine what answers make sense.

- **I can compare properties of two functions graphically, in table form, and algebraically.**

Students will need to interpret the problem situation to determine what answers make sense.

- **I can find key features of a graph using a graph, a table, or an equation.**

Students would need to be familiar with the definitions of key terms.

- **I can state the appropriate domain of a function that represents a problem situation.**

Students would need to be familiar with the definitions of key terms.

Instructional Resources:

Glencoe McGraw Hill Algebra: Concepts and Applications © 2008 parts of Chapter 6

- **I can determine if a relation is a function.**

http://mathtalesfromthespring.blogspot.com/2012/06/determining-whether-relation-is.html
http://everybodyisageniusblog.blogspot.com/2012/12/teaching-functions.html
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module5-5.html

- **I can evaluate functions using function notation.**

http://everybodyisageniusblog.blogspot.com/2013/03/function-machines.html
http://mathequalslove.blogspot.com/2014/02/fabulous-function-machines.html
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module5-3.html

- **I can convert a list of numbers (a sequence) into a function.**

http://www.insidemathematics.org/problems-of-the-month/pom-tritriangles.pdf
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module5-6.html

- **I can graph an equation on the coordinate plane using a table.**

http://www.insidemathematics.org/common-core-math-tasks/high-school/HS-A-2006%20Graphs2006.pdf
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module5-4.html

- **I can compare properties of two functions graphically, in table form, and algebraically.**

http://www.insidemathematics.org/common-core-math-tasks/high-school/HS-F-2007%20Graphs2007.pdf

- **I can find key features of a graph using a graph, a table, or an equation.**

http://squarerootofnegativeoneteachmath.blogspot.com/2013/12/domain-range-from-graph.html

- **I can state the appropriate domain of a function that represents a problem situation.**

http://simplifyingradicals2.blogspot.com/2014/02/domain-and-range-nailed-it.html

Assessment:

Formative:

- exit slips
- observation of students working on problems in class
- observation of student work from outside of class practice problems

Summative:

- unit test questions

Assessment Resources:

http://www.illustrativemathematics.org/
http://practice.parcc.testnav.com/
IIS
Glencoe McGraw Hill Algebra: Concepts and Applications © 2008

Unit 7 – Linear Functions

(4 ½ weeks, February/March)

Common Core State Standards Addressed:

F-IF.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
F-IF.7a: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* Graph linear and quadratic functions and show intercepts, maxima, and minima.
F-BF.1a: Write a function that describes a relationship between two quantities.*
F-LE.1a: Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
F-LE.1b: Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
F-LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
F-LE.5: Interpret the parameters in a linear or exponential function in terms of a context.
S-ID.7: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Student Friendly Learning Targets (In order of teaching):

- I can calculate and interpret the average rate of change of a function.
- I can graph a linear function and identify its intercepts.
- I can demonstrate that a linear function has a constant rate of change.
- I can identify situations in which one quantity changes at a constant rate.
- I can construct linear functions from an arithmetic sequence, graph, table of values, or description of the relationship.
- I can interpret the parameters of a linear function in a real-life problem.

Vocabulary:

function, rate of change, average rate of change, interval, slope, evaluate, domain, range, input, output, equation, x-intercept, y-intercept, linear function, coordinate plane, linear function, arithmetic sequence, linear equation, linear model, units, data

Materials and/or Technology Needed:

calculator, graphing calculator, Desmos.com, graph paper, ruler

Instructional Strategies:

Information from all Learning Targets can be found at Ohio's Model Curricula: http://education.ohio.gov/Topics/Academic-Content-Standards/Mathematics and KCTM's Flip

Book (which encompasses several states' information including Ohio):
http://www.katm.org/baker/pages/common-core-resources.php

- **I can calculate and interpret the average rate of change of a function.**

(From Ohio's Model Curricula) Given a table of values, such as the height of a plant over time, students can estimate the rate of plant growth. Also, if the relationship between time and height is expressed as a linear equation, students should explain the meaning of the slope of the line. Finally, if the relationship is illustrated as a linear or non-linear graph, the student should select points on the graph and use them to estimate the growth rate over a given interval.

The key is that two quantitative variables are being measured on the same subject. The paired data should be listed and then displayed in a scatterplot. If time is one of the variables, it usually goes on the horizontal axis. That which is being predicted goes on the vertical; the predictor variable is on the horizontal axis.

Note that unlike a two-dimensional graph in mathematics, the scales of a scatterplot need not be the same, and even if they are similar (such as SAT Math and SAT Verbal), they still need not have the same spacing. So, visual rendering of slope makes no sense in most scatterplots, i.e., a 45 degree line on a scatterplot need not mean a slope of 1.

Often the interpretation of the intercept (constant term) is not meaningful in the context of the data. For example, this is the case when the zero point on the horizontal is of considerable distance from the values of the horizontal variable, or in some case has no meaning such as for SAT variables.

<u>Common Misconceptions (from Ohio's Model Curricula):</u>
Students may also believe that the slope of a linear function is merely a number used to sketch the graph of the line. In reality, slopes have real-world meaning, and the idea of a rate of change is fundamental to understanding major concepts from geometry to calculus.

Students may believe that a 45 degree line in the scatterplot of two numerical variables always indicates a slope of 1 which is the case only when the two variables have the same scaling. Because the scaling for many real-world situation varies greatly students need to be give opportunity to compare graphs of differing scale. Asking students questions like; What would this graph look like with a different scale or using this scale? Is essential in addressing this misconception.

- **I can graph a linear function and identify its intercepts.**

Use various representations of the same function to emphasize different characteristics of that function.

<u>Common Misconceptions (from Ohio's Model Curricula):</u>

Additionally, students may believe that the process of rewriting equations into various forms is simply an algebra symbol manipulation exercise, rather than serving a purpose of allowing different features of the function to be exhibited.

- **I can demonstrate that a linear function has a constant rate of change.**

(From Ohio's Model Curricula) Compare tabular representations of a variety of functions to show that linear functions have a first common difference (i.e., equal differences over equal intervals), while exponential functions do not (instead function values grow by equal factors over equal x-intervals).

Apply linear and exponential functions to real-world situations. For example, a person earning $10 per hour experiences a constant rate of change in salary given the number of hours worked, while the number of bacteria on a dish that doubles every hour will have equal factors over equal intervals.

Provide examples of arithmetic and geometric sequences in graphic, verbal, or tabular forms, and have students generate formulas and equations that describe the patterns.

Use a graphing calculator or computer program to compare tabular and graphic representations of exponential and polynomial functions to show how the y (output) values of the exponential function eventually exceed those of polynomial functions.

Common Misconceptions (from Ohio's Model Curricula):
Students may believe that all functions have a first common difference and need to explore to realize that, for example, a quadratic function will have equal second common differences in a table.

- **I can identify situations in which one quantity changes at a constant rate.**

(From Ohio's Model Curricula) Compare tabular representations of a variety of functions to show that linear functions have a first common difference (i.e., equal differences over equal intervals), while exponential functions do not (instead function values grow by equal factors over equal x-intervals).

Apply linear and exponential functions to real-world situations. For example, a person earning $10 per hour experiences a constant rate of change in salary given the number of hours worked, while the number of bacteria on a dish that doubles every hour will have equal factors over equal intervals.

Provide examples of arithmetic and geometric sequences in graphic, verbal, or tabular forms, and have students generate formulas and equations that describe the patterns.

Use a graphing calculator or computer program to compare tabular and graphic representations of exponential and polynomial functions to show how the y (output) values of the exponential function eventually exceed those of polynomial functions.

Common Misconceptions (from Ohio's Model Curricula):
Students may believe that all functions have a first common difference and need to explore to realize that, for example, a quadratic function will have equal second common differences in a table.

- **I can construct linear functions from an arithmetic sequence, graph, table of values, or description of the relationship.**

(From Ohio's Model Curricula) Provide a real-world example (e.g., a table showing how far a car has driven after a given number of minutes, traveling at a uniform speed), and examine the table by looking "down" the table to describe a recursive relationship, as well as "across" the table to determine an explicit formula to find the distance traveled if the number of minutes is known.

Write out terms in a table in an expanded form to help students see what is happening. For example, if the *y*-values are 2, 4, 8, 16, they could be written as 2, 2(2), 2(2)(2), 2(2)(2)(2), etc., so that students recognize that 2 is being used multiple times as a factor.

Common Misconceptions (from Ohio's Model Curricula):
Students may believe that the best (or only) way to generalize a table of data is by using a recursive formula. Students naturally tend to look "down" a table to find the pattern but need to realize that finding the 100th term requires knowing the 99th term unless an explicit formula is developed.

Students may also believe that arithmetic and geometric sequences are the same. Students need experiences with both types of sequences to be able to recognize the difference and more readily develop formulas to describe them.

- **I can interpret the parameters of a linear function in a real-life problem.**

(From Ohio's Model Curricula) Use real-world contexts to help students understand how the parameters of linear and exponential functions depend on the context. For example, a plumber who charges \$50 for a house call and \$85 per hour would be expressed as the function $y = 85x + 50$, and if the rate were raised to \$90 per hour, the function would become $y = 90x + 50$. On the other hand, an equation of $y = 8{,}000(1.04)^x$ could model the rising population of a city with 8,000 residents when the annual growth rate is 4%. Students can examine what would happen to the population over 25 years if the rate were 6% instead of 4% or the effect on the equation and function of the city's population were 12,000 instead of 8,000.

Graphs and tables can be used to examine the behaviors of functions as parameters are changed, including the comparison of two functions such as what would happen to a population if it grew by 500 people per year, versus rising an average of 8% per year over the course of 10 years.

Common Misconceptions (from Ohio's Model Curricula):
Students may believe that changing the slope of a linear function from "2" to "3" makes the graph steeper without realizing that there is a real-world context and reason for examining the slopes of lines. Similarly, an exponential function can appear to be abstract until applying it to a real-world situation involving population, cost, investments, etc.

Strategies for Diverse Learners:
Problems of varying difficulty are incorporated into the problem sets.

Literacy Standards Considerations:

- **I can calculate and interpret the average rate of change of a function.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can graph a linear function and identify its intercepts.**

Students would need to be familiar with the definitions of key terms.

- **I can demonstrate that a linear function has a constant rate of change.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can identify situations in which one quantity changes at a constant rate.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can construct linear functions from an arithmetic sequence, graph, table of values, or description of the relationship.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can interpret the parameters of a linear function in a real-life problem.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

Instructional Resources:
Glencoe McGraw Hill Algebra: Concepts and Applications © 2008 parts of Chapter 7

- **I can calculate and interpret the average rate of change of a function.**

https://www.youtube.com/watch?v=avS6C6_kvXM
http://xypi.wordpress.com/2013/07/02/rate-of-change-cards/
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module6-1.html

- **I can graph a linear function and identify its intercepts.**

http://www.insidemathematics.org/common-core-math-tasks/high-school/HS-A-2006%20Graphs2006.pdf
http://www.teachesmath.com/?p=507
http://typeamathland.blogspot.com/2014/02/making-graphing-practice-more.html
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module6-3.html
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module6-5.html

- **I can demonstrate that a linear function has a constant rate of change.**

http://everybodyisageniusblog.blogspot.com/2013/01/patterns.html
http://exponentialcurve.blogspot.com/2013/09/linear-patterns-in-algebra-1.html

- **I can identify situations in which one quantity changes at a constant rate.**

http://www.insidemathematics.org/common-core-math-tasks/high-school/HS-F-2008%20Functions.pdf

- **I can construct linear functions from an arithmetic sequence, graph, table of values, or description of the relationship.**

http://mathcoachblog.com/2014/03/10/linear-function-stories/
http://graphingstories.com/
http://justtellmetheanswer.wordpress.com/2013/11/06/pizza-functions/
http://blog.mrmeyer.com/2008/linear-fun-2-stacking-cups/
http://everybodyisageniusblog.blogspot.com/2013/02/linear-equations-applications.html
http://www.insidemathematics.org/problems-of-the-month/pom-tritriangles.pdf
http://www.visualpatterns.org
http://www.illustrativemathematics.org/illustrations/243
http://simplifyingradicals2.blogspot.com/2013/11/code-crackers.html
http://simplifyingradicals2.blogspot.com/2013/11/domino-effect-from-mathalicious.html
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module6-2.html
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module6-4.html

- **I can interpret the parameters of a linear function in a real-life problem.**

http://untilnextstop.blogspot.com/2010/10/activities-to-help-kids-understand.html

Assessment:
Formative:

- exit slips
- observation of students working on problems in class
- observation of student work from outside of class practice problems

Summative:

- unit test questions

Assessment Resources:

http://www.illustrativemathematics.org/
http://practice.parcc.testnav.com/
IIS
Glencoe McGraw Hill Algebra: Concepts and Applications © 2008

Unit 8 – Probability

(2 weeks, March/April)

Common Core State Standards Addressed:

7-SP.5: Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
7-SP.8a: Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

Student Friendly Learning Targets (In order of teaching):

- I can find the probability of a simple event.
- I can find the odds of a simple event.
- I can find the probability of mutually exclusive events.
- I can find the probability of inclusive events.

Vocabulary:

probability, event, sample space, odds, mutually exclusive events, inclusive events

Materials and/or Technology Needed:

calculator

Instructional Strategies:

Information from all Learning Targets can be found at Ohio's Model Curricula: http://education.ohio.gov/Topics/Academic-Content-Standards/Mathematics and KCTM's Flip Book (which encompasses several states' information including Ohio): http://www.katm.org/baker/pages/common-core-resources.php

- **I can find the probability of a simple event.**
- **I can find the odds of a simple event.**
- **I can find the probability of mutually exclusive events.**
- **I can find the probability of inclusive events.**

(From Ohio's Model Curricula) Grade 7 is the introduction to the formal study of probability. Through multiple experiences, students begin to understand the probability of chance (simple and compound), develop and use sample spaces, compare experimental and theoretical probabilities, develop and use graphical organizers, and use information from simulations for predictions.

Help students understand the probability of chance is using the benchmarks of probability: 0, 1 and ½. Provide students with situations that have clearly defined probability of never happening as zero, always happening as 1 or equally likely to happen as to not happen as 1/2. Then advance to situations in which the probability is somewhere between any two of these benchmark values. This builds to the concept of expressing the probability as a number between 0 and 1. Use this to build the understanding that the closer the probability is to 0, the more likely it will not happen, and the closer to 1, the more likely it will happen. Students learn to make predictions about the relative frequency of an event by using simulations to collect, record, organize and analyze data. Students also develop the understanding that the more the simulation for an event is repeated, the closer the experimental probability approaches the theoretical probability.

Have students develop probability models to be used to find the probability of events. Provide students with models of equal outcomes and models of not equal outcomes are developed to be used in determining the probabilities of events.

Students should begin to expand the knowledge and understanding of the probability of simple events, to find the probabilities of compound events by creating organized lists, tables and tree diagrams. This helps students create a visual representation of the data; i.e., a sample space of the compound event. From each sample space, students determine the probability or fraction of each possible outcome. Students continue to build on the use of simulations for simple probabilities and now expand the simulation of compound probability.

Providing opportunities for students to match situations and sample spaces assists students in visualizing the sample spaces for situations.

Students often struggle making organized lists or trees for a situation in order to determine the theoretical probability. Having students start with simpler situations that have fewer elements enables them to have successful experiences with organizing lists and trees diagrams. Ask guiding questions to help students create methods for creating organized lists and trees for situations with more elements.

Students often see skills of creating organized lists, tree diagrams, etc. as the end product. Provide students with experiences that require the use of these graphic organizers to determine the theoretical probabilities. Have them practice making the connections between the process of creating lists, tree diagrams, etc. and the interpretation of those models.

Additionally, students often struggle when converting forms of probability from fractions to percents and vice versa. To help students with the discussion of probability, don't allow the symbol manipulation/conversions to detract from the conversations. By having students use technology such as a graphing calculator or computer software to simulate a situation and graph the results, the focus is on the interpretation of the data. Students then make predictions about the general population based on these probabilities.

<u>Strategies for Diverse Learners:</u>
Problems of varying difficulty are incorporated into the problem sets.

<u>Literacy Standards Considerations:</u>

- **I can find the probability of a simple event.**

Students would need to interpret the situation to determine which units may be needed to change to the answer required.

- **I can find the odds of a simple event.**

Students would need to be familiar with the definitions of key terms.

- **I can find the probability of mutually exclusive events.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can find the probability of inclusive events.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

Instructional Resources:

Glencoe McGraw Hill Algebra: Concepts and Applications © 2008 parts of Chapter 5

- **I can find the probability of a simple event.**

https://mathymcmatherson.wordpress.com/2014/03/25/clarifying-expectations/

- **I can find the odds of a simple event.**

- **I can find the probability of mutually exclusive events.**

- **I can find the probability of inclusive events.**

Assessment:

Formative:

- exit slips
- observation of students working on problems in class
- observation of student work from outside of class practice problems

Summative:

- unit test questions

Assessment Resources:

http://www.illustrativemathematics.org/
http://practice.parcc.testnav.com/
IIS
Glencoe McGraw Hill Algebra: Concepts and Applications © 2008

Unit 9 – Statistical Models

(3 weeks, April/May)

Common Core State Standards Addressed:

N-Q.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
N-Q.2: Define appropriate quantities for the purpose of descriptive modeling.
N-Q.3: Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
S-ID.1: Represent data with plots on the real number line (dot plots, histograms, and box plots).
S-ID.2: Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
S-ID.3: Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
S-ID.5: Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

Student Friendly Learning Targets (In order of teaching):

- I can describe the center of the data distribution (mean or median).
- I can describe the spread of the data distribution (interquartile range or standard deviation).
- I can represent data with plots on the real number line (dot plots, histograms, and box plots).
- I can compare the distribution of two or more data sets by examining their shapes, centers, and spreads when drawn on the same scale, and can account for effects of extreme data points.
- I can read and interpret the data displayed in a two-way frequency table.
- I can interpret and explain the meaning of relative frequencies in the context of a problem.

Vocabulary:

dot plot, histogram, box plot, 5-number summary, median, lower quartile, upper quartile, minimum value, maximum value, data, frequency, interval, scale, distribution, shape, center, spread, mean, interquartile range, standard deviation, data distribution, outlier, two-way frequency table, percentages, ratios, relative frequencies

Materials and/or Technology Needed:

scientific calculator (TI-30XIIS), graphing calculator, graph paper, ruler, Excel

Instructional Strategies:

Information from all Learning Targets can be found at Ohio's Model Curricula: http://education.ohio.gov/Topics/Academic-Content-Standards/Mathematics and KCTM's Flip Book (which encompasses several states' information including Ohio): http://www.katm.org/baker/pages/common-core-resources.php

For all learning targets involving graphing, from the Ohio Model Curricula:
Graphical representations serve as visual models for understanding phenomena that take place in our daily surroundings. The use of different kinds of graphical representations along with their units, labels and titles demonstrate the level of students' understanding and foster the ability to reason, prove, self-check, examine relationships and establish the validity of arguments. Students need to be able to identify misleading graphs by choosing correct units and scales to create a correct representation of a situation or to make a correct conclusion from it.

(From Ohio's Model Curricula) Students need to develop sound mathematical reasoning skills and forms of argument to make reasonable judgments about their solutions. They should be able to decide whether a problem calls for an estimate, for an approximation, or for an exact answer. To accomplish this goal, teachers should provide students with a broad range of contextual problems that offer opportunities for performing operations with quantities involving units. These problems should be connected to science, engineering, economics, finance, medicine, etc.

Students should be able to correctly identify the degree of precision of the answers which should not be far greater than the actual accuracy of the measurements.

For the learning targets on measures of center, measures of spread, and plotting data, from Ohio's Model Curricula:
It is helpful for students to understand that a statistical process is a problem-solving process consisting of four steps: formulating a question that can be answered by data; designing and implementing a plan that collects appropriate data; analyzing the data by graphical and/or numerical methods; and interpreting the analysis in the context of the original question. Opportunities should be provided for students to work through the statistical process. In Grades 6-8, learning has focused on parts of this process. Now is a good time to investigate a problem of interest to the students and follow it through. The richer the question formulated, the more interesting is the process. Teachers and students should make extensive use of resources to perfect this very important first step. Global web resources can inspire projects.

- **I can describe the center of the data distribution (mean or median).**

(From Ohio's Model Curricula) Measures of center and spread for data sets without outliers are the mean and standard deviation, whereas median and interquartile ranges are better measures for data sets with outliers.

- **I can describe the spread of the data distribution (interquartile range or standard deviation).**

(From Ohio's Model Curricula) Measures of center and spread for data sets without outliers are the mean and standard deviation, whereas median and interquartile ranges are better measures for data sets with outliers.

Introduce the formula of standard deviation by reviewing the previously learned MAD (mean absolute deviation). The MAD is very intuitive and gives a solid foundation for developing the more complicated standard deviation measure.

- **I can represent data with plots on the real number line (dot plots, histograms, and box plots).**

(From Ohio's Model Curricula) Have students practice their understanding of the different types of graphs for categorical and numerical variables by constructing statistical posters. Note that a bar graph for categorical data may have frequency on the vertical (student's pizza preferences) or measurement on the vertical (radish root growth over time - days).

Informally observing the extent to which two boxplots or two dotplots overlap begins the discussion of drawing inferential conclusions. Don't shortcut this observation in comparing two data sets.

As histograms for various data sets are drawn, common shapes appear. To characterize the shapes, curves are sketched through the midpoints of the tops of the histogram's rectangles. Of particular importance is a symmetric unimodal curve that has specific areas within one, two, and three standard deviations of its mean. It is called the Normal distribution and students need to be able to find areas (probabilities) for various events using tables or a graphing calculator.

<u>Common Misconceptions (from Ohio's Model Curricula):</u>
<u>Students may believe:</u>
That a bar graph and a histogram are the same. A bar graph is appropriate when the horizontal axis has categories and the vertical axis is labeled by either frequency (e.g., book titles on the horizontal and number of students who like the respective books on the vertical) or measurement of some numerical variable (e.g., days of the week on the horizontal and median length of root growth of radish seeds on the vertical). A histogram has units of measurement of a numerical variable on the horizontal (e.g., ages with intervals of equal length).

That the lengths of the intervals of a boxplot (min,Q1), (Q1,Q2), (Q2,Q3), (Q3,max) are related to the number of subjects in each interval. Students should understand that each interval theoretically contains one-fourth of the total number of subjects. Sketching an accompanying histogram and constructing a live boxplot may help in alleviating this misconception.

- **I can compare the distribution of two or more data sets by examining their shapes, centers, and spreads when drawn on the same scale, and can account for effects of extreme data points.**

- **I can read and interpret the data displayed in a two-way frequency table.**

(From Ohio's Model Curricula) In the categorical case, begin with two categories for each variable and represent them in a two-way table with the two values of one variable defining the rows and the two values of the other variable defining the columns. (Extending the number of rows and columns is easily done once students become comfortable with the 2x2 case.) The table entries are the joint frequencies of how many subjects displayed the respective cross-classified values. Row totals and column totals constitute the marginal frequencies. Dividing joint or marginal frequencies by the total number of subjects define relative frequencies (and percentages), respectively. Conditional relative frequencies are determined by focusing on a specific row or column of the table. They are particularly useful in determining any associations between the two variables.

- **I can interpret and explain the meaning of relative frequencies in the context of a problem.**

(From Ohio's Model Curricula) In the categorical case, begin with two categories for each variable and represent them in a two-way table with the two values of one variable defining the rows and the two values of the other variable defining the columns. (Extending the number of rows and columns is easily done once students become comfortable with the 2x2 case.) The table entries are the joint frequencies of how many subjects displayed the respective cross-classified values. Row totals and column totals constitute the marginal frequencies. Dividing joint or marginal frequencies by the total number of subjects define relative frequencies (and percentages), respectively. Conditional relative frequencies are determined by focusing on a specific row or column of the table. They are particularly useful in determining any associations between the two variables.

Strategies for Diverse Learners:

Problems of varying difficulty are incorporated into the problem sets.

Literacy Standards Considerations:

- **I can describe the center of the data distribution (mean or median).**

Students would need to be familiar with the definitions of key terms.

- **I can describe the spread of the data distribution (interquartile range or standard deviation).**

Students would need to be familiar with the definitions of key terms.

- **I can represent data with plots on the real number line (dot plots, histograms, and box plots).**

Students would need to be familiar with the definitions of key terms.

- **I can compare the distribution of two or more data sets by examining their shapes, centers, and spreads when drawn on the same scale, and can account for effects of extreme data points.**

Students would need to be familiar with the definitions of key terms. Word Problems are included as a part of the practice set - students will have to interpret the key terms as they apply to the problem.

- **I can read and interpret the data displayed in a two-way frequency table.**

Students would need to be familiar with the definitions of key terms. Word Problems are included as a part of the practice set - students will have to interpret the key terms as they apply to the problem.

- **I can interpret and explain the meaning of relative frequencies in the context of a problem.**

Students would need to be familiar with the definitions of key terms. Word Problems are included as a part of the practice set - students will have to interpret the key terms as they apply to the problem.

Instructional Resources:
Glencoe McGraw Hill Algebra: Concepts and Applications © 2008 parts of Chapter 3

- **I can describe the center of the data distribution (mean or median).**

http://www.oconee.k12.sc.us/webpages/thavice/files/statistics%20unit%20lesson%201%20-%20activity%201%20-%20penny%20activity.pdf
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module10-2.html

- **I can describe the spread of the data distribution (interquartile range or standard deviation).**

http://www.oconee.k12.sc.us/webpages/thavice/files/statistics%20unit%20lesson%201%20-%20activity%201%20-%20penny%20activity.pdf
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module10-5.html
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/A-9-1.html

- **I can represent data with plots on the real number line (dot plots, histograms, and box plots).**

http://www.oconee.k12.sc.us/webpages/thavice/files/statistics%20unit%20lesson%201%20-%20activity%201%20-%20penny%20activity.pdf
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module10-3.html

- **I can compare the distribution of two or more data sets by examining their shapes, centers, and spreads when drawn on the same scale, and can account for effects of extreme data points.**

- **I can read and interpret the data displayed in a two-way frequency table.**

- **I can interpret and explain the meaning of relative frequencies in the context of a problem.**

Assessment:

Formative:

- exit slips
- observation of students working on problems in class
- observation of student work from outside of class practice problems

Summative:

- unit test questions

Assessment Resources:

http://www.illustrativemathematics.org/
http://practice.parcc.testnav.com/
IIS
Glencoe McGraw Hill Algebra: Concepts and Applications © 2008

Unit 10 – Properties of Exponents

(2 weeks, May)

Common Core State Standards Addressed:

8-EE.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

8-EE.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger*.

8-EE.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

A-APR.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Student Friendly Learning Targets (In order of teaching):

- I can apply the properties of integer exponents.
- I can simplify expressions containing negative exponents.
- I can express numbers in scientific notation.
- I can evaluate expression involving multiplication or division and scientific notation.
- I can identify and classify polynomials and find their degree.
- I can add and subtract polynomials.

Vocabulary:

exponent, power, scientific notation, polynomials, degree, coefficient

Materials and/or Technology Needed:

calculator, graph paper, graphing calculator, Desmos.com

Instructional Strategies:

Information from all Learning Targets can be found at Ohio's Model Curricula: http://education.ohio.gov/Topics/Academic-Content-Standards/Mathematics and KCTM's Flip Book (which encompasses several states' information including Ohio): http://www.katm.org/baker/pages/common-core-resources.php

- **I can apply the properties of integer exponents.**
- **I can simplify expressions containing negative exponents.**
- **I can express numbers in scientific notation.**

- **I can evaluate expression involving multiplication or division and scientific notation.**

(From Ohio's Model Curricula for all four) Although students begin using whole-number exponents in Grades 5 and 6, it is in Grade 8 when students are first expected to know and use the properties of exponents and to extend the meaning beyond counting-number exponents. It is no accident that these expectations are simultaneous, because it is the properties of counting-number exponents that provide the rationale for the properties of integer exponents. In other words, s*tudents should not be told these properties but rather should derive them through experience and reason.*

For counting-number exponents (and for nonzero bases), the following properties follow directly from the meaning of exponents.

1. $a^n a^m = a^{n+m}$
2. $(a^n)^m = a^{nm}$
3. $a^n b^n = (ab)^n$

Students should have experience simplifying numerical expressions with exponents so that these properties become natural and obvious. For example,

$$2^3 \cdot 2^5 = (2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) = 2^8$$

$$(5^3)^4 = (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5) = 5^{12}$$

$$(3 \cdot 7)^4 = (3 \cdot 7) \cdot (3 \cdot 7) \cdot (3 \cdot 7) \cdot (3 \cdot 7) = (3 \cdot 3 \cdot 3 \cdot 3) \cdot (7 \cdot 7 \cdot 7 \cdot 7) = 3^4 \cdot 7^4$$

If students reason about these examples with a sense of generality about the numbers, they begin to articulate the properties. For example, "I see that 3 twos is being multiplied by 5 twos, and the results is 8 twos being multiplied together, where the 8 is the sum of 3 and 5, the number of twos in each of the original factors. That would work for a base other than two (as long as the bases are the same)."

Note: When talking about the meaning of an exponential expression, it is easy to say (incorrectly) that "3^5 means 3 multiplied by itself 5 times." But by writing out the meaning, $3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$, students can see that there are only 4 multiplications. So a better description is "3^5 means 5 3s multiplied together."

Students also need to realize that these simple descriptions work only for counting-number exponents. When extending the meaning of exponents to include 0 and negative exponents, these descriptions are limiting: Is it sensible to say "3^0 means 0 3s multiplied together" or that "3^{-2} means -2 3s multiplied together"?

The motivation for the meanings of 0 and negative exponents is the following principle: *The properties of counting-number exponents should continue to work for integer exponents.*

Properties of Integer Exponents

For any nonzero real numbers a and b and integers n and m:

1. $a^n a^m = a^{n+m}$
2. $(a^n)^m = a^{nm}$
3. $a^n b^n = (ab)^n$
4. $a^0 = 1$
5. $a^{-n} = 1/a^n$

Patterns in Exponents

⋮	⋮
5^4	625
5^3	125
5^2	25
5^1	5
5^0	1
5^{-1}	1/5
5^{-2}	1/25
5^{-3}	1/125
⋮	⋮

As the exponent decreases by 1, the value of the expression is divided by 5, which is the

For example, Property 1 can be used to reason what 3^0 should be. Consider the following expression and simplification: $3^0 \cdot 3^5 = 3^{0+5} = 3^5$. This computation shows that the when 3^0 is multiplied by 3^5, the result (following Property 1) should be 3^5, which implies that 3^0 must be 1. Because this reasoning holds for any base other than 0, we can reason that $a^0 = 1$ for any nonzero number a.

To make a judgment about the meaning of 3^{-4}, the approach is similar: $3^{-4} \cdot 3^4 = 3^{-4+4} = 3^0 = 1$. This computation shows that 3^{-4} should be the reciprocal of 3^4, because their product is 1. And again, this reasoning holds for any nonzero base. Thus, we can reason that $a^{-n} = 1/a^n$.

Putting all of these results together, we now have the properties of integer exponents, shown in the above chart. For mathematical completeness, one might prove that properties 1-3 continue to hold for integer exponents, but that is not necessary at this point.

A supplemental strategy for developing meaning for integer exponents is to make use of patterns, as shown in the chart to the right.

The meanings of 0 and negative-integer exponents can be further explored in a place-value chart:

thousands	*hundreds*	*tens*	*ones*		*tenths*	*hundredths*	*thousandths*
10^3	10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}
3	2	4	7	.	5	6	8

Thus, integer exponents support writing any decimal in expanded form like the following:

$$3247.568 = 3 \cdot 10^3 + 2 \cdot 10^2 + 4 \cdot 10^1 + 7 \cdot 10^0 + 5 \cdot 10^{-1} + 6 \cdot 10^{-2} + 8 \cdot 10^{-3}.$$

Expanded form and the connection to place value is important for helping students make sense of scientific notation, which allows very large and very small numbers to be written concisely, enabling easy comparison. To develop familiarity, go back and forth between standard notation and scientific notation for numbers near, for example, 10^{12} or 10^{-9}. Compare numbers, where one is given in scientific notation and the other is given in standard notation. Real-world problems can help students compare quantities and make sense about their relationship.

Provide practical opportunities for students to flexibly move between forms of squared and cubed numbers. For example, If $3^2 = 9$ then $\sqrt{9} = 3$. This flexibility should be experienced symbolically and verbally.

Common Misconceptions from Ohio's Model Curricula:
Students may mix up the product of powers property and the power of a power property. Is $x^2 \cdot x^3$ equivalent to x^5 or x^6? Students may make the relationship that in scientific notation, when a number contains one nonzero digit and a positive exponent, that the number of zeros equals the exponent. This pattern may incorrectly be applied to scientific notation values with negative values or with more than one nonzero digit.

- **I can identify and classify polynomials and find their degree.**
- **I can add and subtract polynomials.**

(From Ohio's Model Curricula) The primary strategy for this cluster is to make connections between arithmetic of integers and arithmetic of polynomials. In order to understand this standard, students need to work toward both understanding and fluency with polynomial arithmetic. Furthermore, to talk about their work, students will need to use correct vocabulary, such as integer, monomial, polynomial, factor, and term.

In arithmetic of polynomials, a central idea is the distributive property, because it is fundamental not only in polynomial multiplication but also in polynomial addition and subtraction. With the distributive property, there is little need to emphasize misleading mnemonics, such as FOIL, which is relevant only when multiplying two binomials, and the procedural reminder to "collect like terms" as a consequence of the distributive property. For example, when adding the polynomials 3x and 2x, the result can be explained with the distributive property as follows: 3x + 2x = (3 + 2)x = 5x.

Now for polynomials, students need to reason that the sum (difference or product) of any two polynomials is indeed a polynomial. At first, restrict attention to polynomials with integer coefficients. Later, students should consider polynomials with rational or real coefficients and reason that such polynomials are closed under these operations.

Literacy Standards Considerations:

- **I can apply the properties of integer exponents.**

Students would need to be familiar with the definitions of key terms.

- **I can simplify expressions containing negative exponents.**

Students would need to be familiar with the definitions of key terms.

- **I can express numbers in scientific notation.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can evaluate expression involving multiplication or division and scientific notation.**

- **I can identify and classify polynomials and find their degree.**

Students would need to be familiar with the definitions of key terms.

- **I can add and subtract polynomials.**

Students would need to be familiar with the definitions of key terms.

Strategies for Diverse Learners:

Problems of varying difficulty are incorporated into the problem sets.

Instructional Resources:

Glencoe McGraw Hill Algebra: Concepts and Applications © 2008 parts of Chapters 8 and 9

- **I can apply the properties of integer exponents.**

http://dontpanictheansweris42.blogspot.com/2013/08/exponent-rules-game.html
http://www.hoppeninjamath.com/teacherblog/?p=361
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module7-1.html
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module7-2.html

- **I can simplify expressions containing negative exponents.**

http://dontpanictheansweris42.blogspot.com/2013/08/exponent-rules-game.html
http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module7-2.html

- **I can express numbers in scientific notation.**

http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module7-3.html

- **I can evaluate expression involving multiplication or division and scientific notation.**

http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module7-3.html

- **I can identify and classify polynomials and find their degree.**

http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module7-4.html

- **I can add and subtract polynomials.**

http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module7-5.html

Assessment:

Formative:

- exit slips
- observation of students working on problems in class
- observation of student work from outside of class practice problems

Summative:

- unit test questions

Assessment Resources:

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